A Large Markovian Linear Program Model for Dairy Herd Decision-Making

Victor E. Cabrera

Assistant Professor
Extension Dairy Specialist
Department of Dairy Science
University of Wisconsin-Madison

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Introduction

- The complex multi-component dairy herd system requires continued economic optimization investigation
- Dynamic programming (**DP**) is the most recognized method to optimize dairy herd economics
- DP formulation becomes easily large and complicated
- Not many end-user applications based on traditional DP formulation

Introduction

- **DP formulation**:
  - **Policy** is the sequence of decisions taken at different **stages**
  - **States** are the various possible conditions in which the system might be
  - **Objective Function** defined for each stage is the value of a function for that stage and all subsequent stages
  - **Solution** usually involve code-writing and value or policy iteration

$$f_n(S_n) = \text{Optimal}\{B_n(X_n)+f_{n+1}(S_{n+1})\} \qquad \text{for } n=N\ldots1$$

Bellman, 1957

Introduction

- An appealing method that simplifies the search for optimal policy is using linear programming (**LP**) to solve DP problems
- LP formulation of DP could have some benefits:
  - Simpler formulation of the problem to be solved
  - Interaction of herd mates in the solution
  - Solution for sub-optimal conditions
  - Flexible handle of time steps
  - Hopefully better interaction with the model
  - Can help to reduce the gap between model construction and potential end-user applications

Objective

- Propose an innovative optimization framework using Markovian linear programming to optimize dairy farm returns under different decision schemes

- Illustrate the model with a practical application studying five diets for entire lactations

Materials and Methods

- Production Scenario
  - States
    - Parity: PAR = 1 to 15
    - Month after calving: MIL = 1 to 24
    - Pregnancy: PREG = 0, 1 to 9
  - Size: 15 x 24 x 10 = 3,600 possible states
  - However MIL > PREG + 2, -54 states per PAR
  - Effective number of states: 2,790

Materials and Methods

- LP objective function

$$\max \sum_{i=1}^{2790} \sum_{k=1}^{2} y_{ik} NR_{ik} \quad [1]$$

- *i* is the state and *k* is the decision to be made (1 = keep and 2 = replace).
- Y_{ik} is the steady state proportion of state *i* when *k* decision is made
- NR_{ik} is the net return expected for the steady state proportion of state *i* when *k* decision is made.

Materials and Methods

- LP constraints

$$y_{ik} \geq 0 \text{ for all } i \text{ and } k \quad [2]$$

$$\sum_{i=1}^{2790} \sum_{k=1}^{2} y_{ik} = 1 \quad [3]$$

$$\sum_{k=1}^{2} y_{ik} - \sum_{i=1}^{2970} \sum_{k=1}^{2} y_{ik} P_{ij}(k) = 0 \text{ for } j = 1 \text{ to } 2{,}790 \quad [4]$$

- $P_{ij}(k)$ is the element of the transition matrix resulting from making decision k

Materials and Methods

- Net return calculation (*k* = 1, keep)

$$NR_{i1} = IOFC_i + INB_i - CDC_i - CIC_i - AI_i + EnvFactor_i \; for \; i = 1 \; to \; 2790 \quad [5]$$

- *IOFC* is income over feed cost
- *INB* is income because of a new born
- *CDC* is cost of a dead cow
- *CIC* is cost of involuntary culling
- *AI* is cost of insemination
- *EnvFactor* is the environmental cost

Materials and Methods

- Net return calculation (*k* = 2, replace)

$$NR_{i2} = SV - (HRC - INB) \; for \; i = 1 \; to \; 2790 \quad [6]$$

- *SV* is salvage value
- *HRC* is heifer replacement cost (bred)
- *INB* is income because of a new born

Materials and Methods

- The *IOFC* is the difference between milk value (*Mv*) and feed cost (*Fc*).

$$IOFC_{i1} = Mv_i - Fc_i = MP_i * Mp - DMI_i * (F\% * Fp + C\% * Cp + SBM\% * SBMp)$$

for *i*=1 to 2970 [7]

- Milk/DMI based on Tessmann et al. (1991) formulated on alfalfa silage (F), high moisture ear corn (C) and soybean meal (SBM)

Materials and Methods

- The *INB* is value of a new born calculated as a weighted average of the probability of being heifer (*PHC*) and bull (1-*PHC*) calf

$$INB_{i1} = PHC * HCp + (1 - PHC) * BCp \quad \text{for } i\text{=1 to 2970 and PREG=9} \quad [8]$$

- *HCp* is the value of a heifer calf and *BCp* is the value of a bull calf

Materials and Methods

- The *CDC* is the composite cost of disposal and replacement of a dead cow with a pregnant, ready-to-deliver heifer

$$CDC_{i1} = Mr_i * (Dc + HRc - INB_i) \qquad \text{for } i\text{=1 to 2970} \quad [9]$$

- *Mr* is mortality rate and *Dc* is disposal cost
- The cost is partially offset by the value of a new born coming with the replacement

Materials and Methods

- The *CIC* is the composite cost of replacing a cow with a pregnant, ready-to-deliver heifer

$$CIC_{i1} = ICr * ((HRc - INB_i) - Sv) \qquad \text{for } i\text{=1 to 2970} \quad [10]$$

- *ICr* is the involuntary culling rate
- The cost is partially offset by the salvage value and the value of a new born coming with the replacement

Materials and Methods

- The *AI* is calculated as the monthly estimated cost of a common reproductive program using artificial insemination including labor, semen, and diagnosis

- Charged to open cows in reproductive status (PREG = 0 & MIL ≥ 2)

Materials and Methods

- The *EnvFactor* is the calculated value of nutrient excreted (*NutValue*) less the cost of manure disposal (*CMD*).

$$EnvFactor_{i1} = CMD_i - NutValue_i \qquad \text{for } i\text{=1 to 2970} \quad [11]$$

- The cost of manure disposal is a function of loading, transporting, unloading and incorporating the excreted manure in nearby crop fields

Materials and Methods

- Biological Parameters (transition probabilities)
  - AgSource Cooperative Services (Verona, WI) 326,000 Holstein lactations for 5-yr period
  - Other published sources

Materials and Methods

- Economic factors
  - Baseline 2008 market conditions for Wisconsin

Factor	Value
Milk	\$0.44/kg (\$18.92/cwt)
Replacement (bred heifer)	\$2000
Salvage value culled cow	\$840 (726 kg x \$1.16/kg)
Disposal cost of dead cow	\$100
New born value	\$500 x 0.467 + \$50 x 0.533
Alfalfa silage	\$0.115/kg
High moisture ear corn	\$0.187/kg
Cost of AI	\$20/mo
Soybean meal (48% CP)	\$0.366/kg
Urea (46%N)	\$0.6071/kg

Materials and Methods

- Experimental design: Five diet treatments (%)

Month in lactation (MIL)	1-3	4-7	8-22
Diet 1			
Alfalfa silage	38	48	68
High moisture ear corn	42	40	25
Soybean meal	18	10	5
Diet 2			
Alfalfa silage	48	58	78
High moisture ear corn	34	33	17
Soybean meal	16	7	3
Diet 3			
Alfalfa silage	58	68	88
High moisture ear corn	27	25	9
Soybean meal	13	5	1
Diet 4			
Alfalfa silage	68	88	98
High moisture ear corn	19	9	0
Soybean meal	11	1	0
Diet 5			
Alfalfa silage	98	98	98
High moisture ear corn	0	0	0
Soybean meal	0	0	0

Adapted from Tessmann et al., 1991

Results and Discussion

Optimal Policy

- Always suggested to replace open cows
- Always higher replacement for multiparous
- Replacement @ MIL=11 (primiparous) and 10 (multiparous) for concentrate diets (1-4)
- Replacement @ MIL=12 (primiparous) and 11 (multiparous) for forage diet (5)
- Favorable market conditions called for higher replacement policies

Results and Discussion

Optimal Policy – Unfavorable Mkt. Conditions

Low milk ($0.22/kg [$10/cwt])

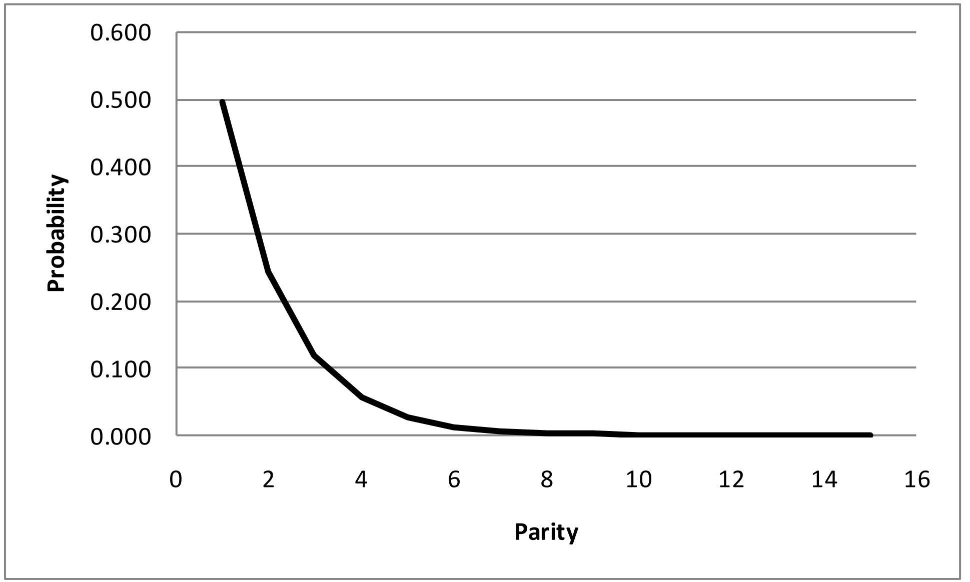
High corn (0.24/kg [$6.1/bu])

- For diets containing concentrates (1-4) replacement @15 MIL (primiparous) and @12 MIL (multiparous)
- For all forage diet (5) replacement @ 15 MIL whether primiparous o multiparous

Results and Discussion

Herd Structure – Diets with Concentrates (1-4)

- As expected, the majority (85.7%) of the population is contained in the first 3 parities. Only 2.8% of animals would be in parity 5, and the proportion of cows reaching parity 10 or higher could be considered negligible



Results and Discussion

MIL	Pregnancy Status 0	1	2	3	4	5	6	7	8
1	0.023785								
2	0.023188								
3	0.017862	0.004801							
4	0.014336	0.003161	0.004703						
5	0.012213	0.002026	0.003103	0.004452					
6	0.010896	0.001338	0.001992	0.002944	0.004268				
7	0.009999	0.000946	0.001318	0.001894	0.002828	0.004142			
8	0.009351	0.000674	0.000934	0.001255	0.001822	0.002749	0.004068		
9	0.008862	0.000497	0.000667	0.000890	0.001209	0.001773	0.002703	0.004011	
10		0.000465	0.000492	0.000637	0.000859	0.001179	0.001747	0.002669	0.003966
11			0.000461	0.000470	0.000615	0.000838	0.001162	0.001726	0.002642
12				0.000441	0.000455	0.000601	0.000827	0.001150	0.001711
13					0.000427	0.000444	0.000593	0.000819	0.001140
14						0.000417	0.000439	0.000588	0.000813
15							0.000413	0.000435	0.000584
16								0.000409	0.000432
17									0.000407
18									
19									
20									
21									
22									
23									
24									

PAR=2, High concentrate diet (1)

Results and Discussion

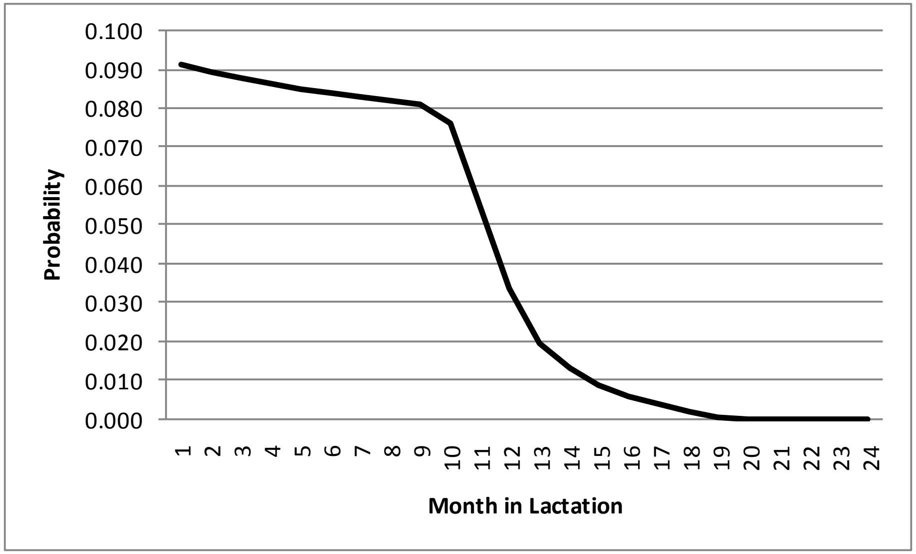
Herd Structure – All Forage Diet (5)

- Optimal structure: 0.482, 0.244, 0.122, 0.061, and 0.061 for parity 1, 2, 3, 4, and 5 to 15, respectively. Again, the majority (84.8%) of the population is contained in the first 3 parities. Only 3.1% of animals would be in parity 5, and less than 0.2% of them will reach parity 10

Results and Discussion

Herd Structure – MIL, Baseline Scenario, Concentrate Diets (1-4)

- Proportion of the herd population decreases because of mortality, involuntary and voluntary culling. About 9.1% of the herd is in first MIL. Only 0.1% is in MIL = 19. No cows reach MIL ≥ 20



Results and Discussion

Market and Constraint Conditions	Diet	N excretion (kg/cow/mo)	Net Revenue ($/cow/mo)
2008 Favorable	1	12.56	132.16
Milk $0.40/kg ($18.2/cwt)	2	12.47	131.79
Corn $0.19/kg ($4.8/bu)	3	12.55	116.92
No N constraint	4	12.09	105.49
	5	11.35	79.84
2008 Unfavorable	1	12.76	-6.63
Milk $0.22/kg ($10/cwt)	2	12.69	-1.40
Corn $0.24/kg ($6.1/bu)	3	12.73	-2.89
No N constraint	4	12.21	0.33
	5	11.54	-1.10
2008 Favorable	1	12.00	119.84
Milk $0.40/kg	2	12.00	126.36
Corn $0.19/kg	3	12.00	104.86
N ≤ 12 kg/mo constraint	4	12.00	104.94
	5	11.35	79.84
2008 Unfavorable	1	12.00	-22.84
Milk $0.22/kg	2	12.00	-8.37
Corn $0.24/kg	3	12.00	-18.02
N ≤ 12 kg/mo constraint	4	12.00	-1.69
	5	11.54	-1.10

Conclusions

- A Markovian LP formulation and solution for DP has not been previously reported for realistic dairy farm conditions

- It complements and adds to the value and policy iteration methods commonly used to solve large DP models

Conclusions

- LP solution of DP problems provides a new set of possibilities in dairy herd cattle decision-making :
  - Interaction among herd mates
  - Sub-optimal solutions
  - Development of decision support systems
  - Flexible handle the length of time steps

Conclusions

- The implementation of a Markovian linear program is an important advancement for dairy decision-making that provides both robustness and versatility in operations research. The model could become a valuable tool for dairy farms to support economic decision-making

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